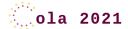
ASKeD-BN: Automatic Synthesis of Boolean Networks from Biological Knowledge and Data



Athénaïs Vaginay, Taha Boukhobza, Malika Smaïl-Tabbone

Jun 21-23 2021



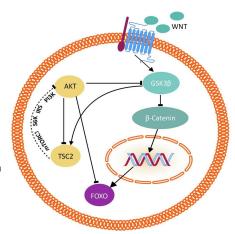




Modeling Biological Systems with Boolean Networks

Boolean Networks (BN) are:

- qualitative formalism, well-suited for biological systems
- built from experimental data and knowledge from literature
- the automatic synthesis of BNs from biological data and knowledge is still a challenge



from Schwab et al. 2020.

$$\mathcal{B} = \begin{cases} f_{\mathsf{A}}: a_{t+1} \ = \ c_t & \text{in the Boolean world: } \mathbb{B} = \{0,1\} \\ f_{\mathsf{B}}: b_{t+1} \ = \ b_t \land \neg c_t & n \text{ Boolean components} \\ f_{\mathsf{C}}: c_{t+1} \ = \ \neg c_t & \mathsf{BN} \ = \text{ set of } n \text{ update functions} \\ & \mathsf{negation: } \neg \quad \mathsf{disjunction: } \lor \quad \mathsf{conjunction: } \land \end{cases}$$

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$$\mathcal{B} = \begin{cases} f_{\mathsf{A}} : a_{t+1} = c_t & \text{status of child component at } t+1 \\ f_{\mathsf{B}} : b_{t+1} = b_t \land \neg c_t & = \mathsf{f}(\mathsf{status of parents components at } t) \\ f_{\mathsf{C}} : c_{t+1} = \neg c_t & \end{cases}$$

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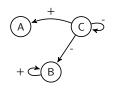
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Boolean Networks - Their Structure and Dynamics

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Boolean Networks – Their Structure and Dynamics

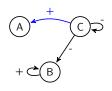


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Interaction Graph (IG)

nodes: components of the BN edges: influences + polarity

Boolean Networks – Their Structure and Dynamics

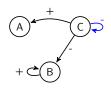


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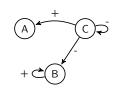


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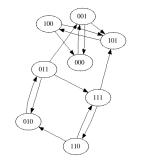
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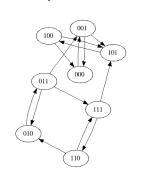


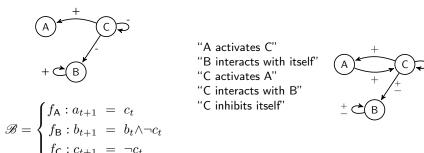
State Transition Graph (STG)

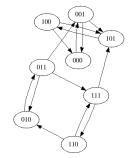
nodes: configurations of the BN (vector $\in \mathbb{B}^n$) e.g. 001, 010, 111, . . .

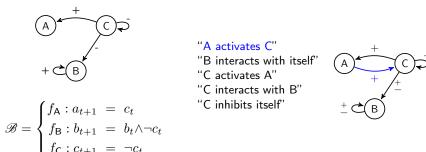
edge from c to c' if c' = f(c)

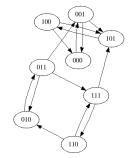


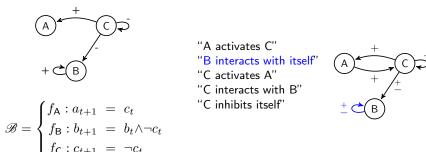


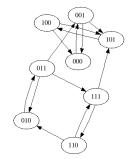


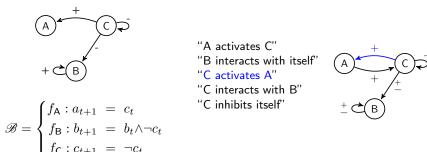


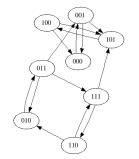


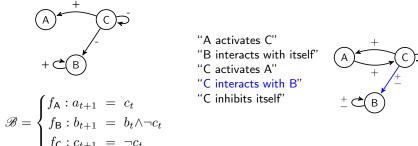


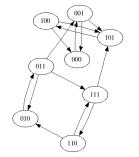


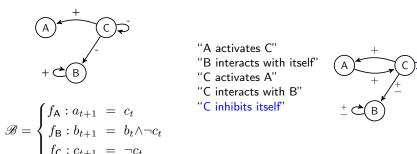


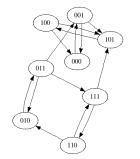


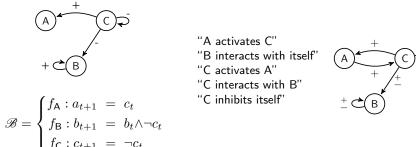


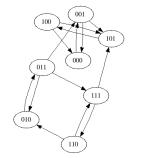


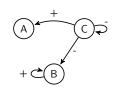




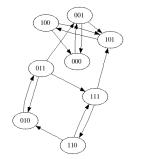








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Prior Knowledge Network (PKN)

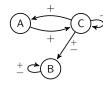
Super-set of influences allowed

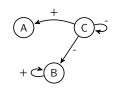
"A activates C"

"B interacts with itself"
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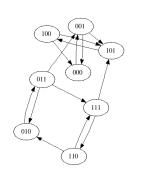
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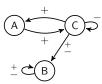


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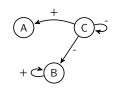
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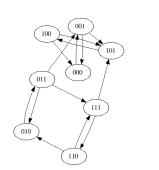


Multivariate Time Series (TS)

t	1	2	3	4	5	6	7	8	9	10	11	12	13	
A	0	3	7	13	20	30	49	61	100	63	36	25	2	
В	0 100	86	64	57	54	53	51	49	45	37	33	28	22	
c	0	27	36	42	60	75	54	44	38	48	60	72	88	



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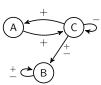


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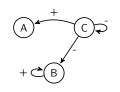
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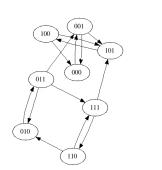


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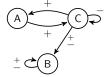


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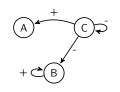
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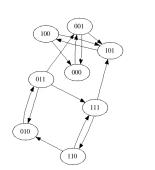


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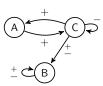


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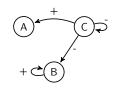
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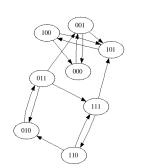


Multivariate Time Series (TS)

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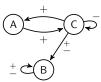
Prior Knowledge Network (PKN)

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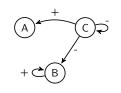


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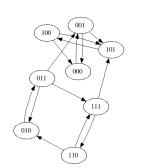
Concentrations of the components over time

1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | . . 0 | 3 | 7 | 13 | 20 | 30 | 49 | 61 | 100 | 63 | 36 | 25 | 2 | . . . | 100 | 86 | 64 | 57 | 54 | 53 | 51 | 49 | 45 | 37 | 33 | 28 | 22 | . . .

C 0 27 36 42 60 75 54 44 38 48 60 72 88



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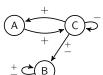
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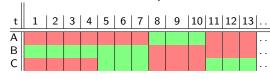
"B interacts with itself"

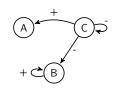
"C activates A"
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"C inhibits itself"

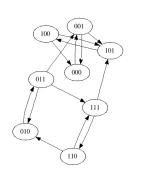


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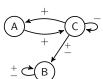
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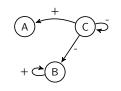
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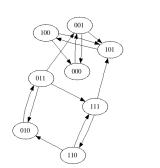


Multivariate Time Series (TS)

t	1	2	3	4	5	6	7	8	9	10	11	12	13	
А														
В														
C														



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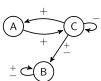
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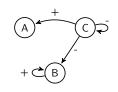
"C interacts with B"

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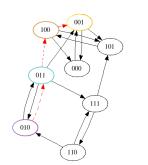


Multivariate Time Series (TS)

			010				$011 \rightarrow 100 \rightarrow 0$							001		
t	1	2	3	4	5	6	7	8	9	10	11	12	13			
A																
В																
C																



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Prior Knowledge Network (PKN)

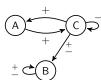
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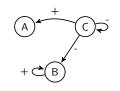
"C inhibits itself"



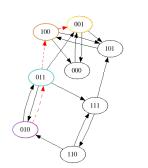
Multivariate Time Series (TS)

		010		\rightarrow	011 →					\rightarrow				
t	1	2	3	4	5	6	7	8	9	10	11	12	13	
A														
В														
C														

Boolean Networks – Synthesis from Knowledge and Data



$$\mathcal{B} = \begin{cases} f_{\mathsf{A}} : a_{t+1} = c_t \\ f_{\mathsf{B}} : b_{t+1} = b_t \land \neg c_t \\ f_{\mathsf{C}} : c_{t+1} = \neg c_t \end{cases}$$



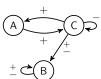
Prior Knowledge Network (PKN)

Super-set of influences allowed

"A activates C" "B interacts with itself" "C activates A"

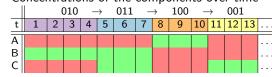
"C interacts with B"

"C inhibits itself"



Multivariate Time Series (TS)

Concentrations of the components over time



Get the best coverage possible

Automatic synthesis of BNs from a Prior Knowledge Network (PKN) and a multivariate Time Series (TS) = hard problem (combinatorial explosion)

REVEAL Best-Fit caspo-TS

Automatic synthesis of BNs from a Prior Knowledge Network (PKN) and a multivariate Time Series (TS) = hard problem (combinatorial explosion)

	PKN	TS	assumptions
REVEAL	unsigned	binarized	time unit $= 1$
Best-Fit	unsigned	binarized	time unit $= 1$
caspo-TS	signed	numeric	local partial-monotony

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Similar principles:

- 1. delimitation of the search space using the PKN as constraint
- 2. optimization of a criteria which measure the adequacy of the synthesized BNs with the TS (coverage)

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Multiple optimal solutions are all returned

Formulation the BN synthesis problem as a logic program with the Answer Set Programming (ASP) framework

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Main parts of the logic program:

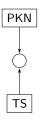
- 1. generates all the possible candidate functions
- 2. removes the ones that do not respect the PKN
- 3. acts like an exhaustive evaluation of all the candidates and returns the *parsimonious* candidates which explain best the *binarized* observations from the given time series

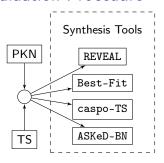
Formulation the BN synthesis problem as a logic program with the Answer Set Programming (ASP) framework

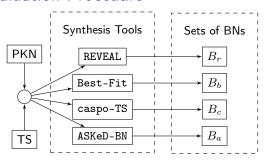
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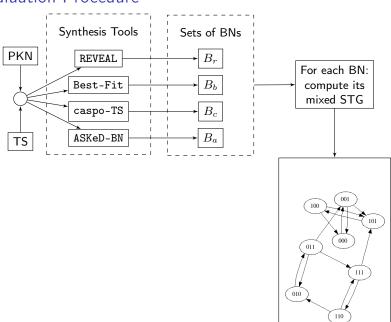
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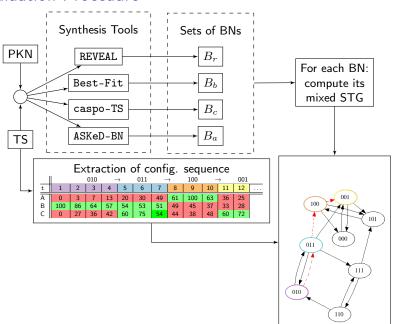
ASP solvers are designed to solve hard combinatorial satisfaction problem. They prune the search space *efficiently* (heuristic from SAT solvers).

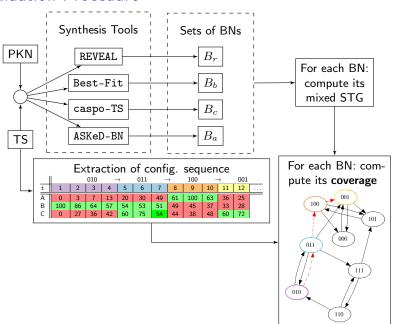


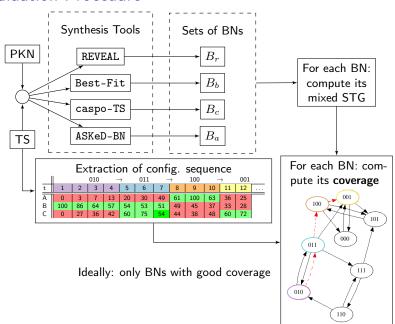












Evaluation on Real Datasets yeast

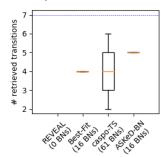
A. thaliana

Evaluation on Real Datasets yeast

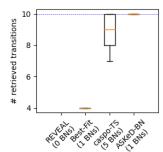
4 components, 7 transitions

A. thaliana 5 components, 10 transitions

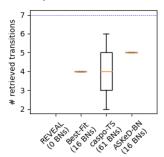
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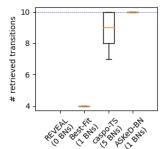


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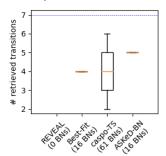


► REVEAL fails

A. thaliana 5 components, 10 transitions

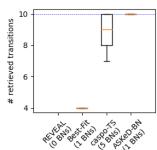


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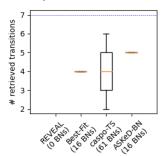


- REVEAL fails
- Best-Fit lacks consistency

A. thaliana5 components, 10 transitions

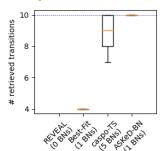


yeast
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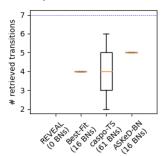


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- ► Best-Fit lacks consistency
- caspo-TS and ASKeD-BN find good BNs

A. thaliana 5 components, 10 transitions

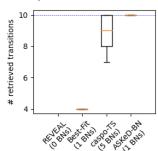


yeast4 components, 7 transitions

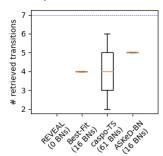


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A. thaliana 5 components, 10 transitions

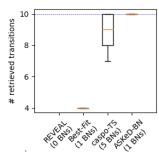


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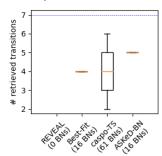
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confirmed on synthetic datasets

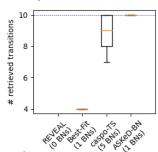
yeast4 components, 7 transitions



- REVEAL fails
- ► Best-Fit lacks consistency
- caspo-TS and ASKeD-BN find good BNs
- caspo-TS returns more BNs, some of them with poor coverage.

→ ASKeD-BN returns a small number of BN, with good coverage and low variance

A. thaliana 5 components, 10 transitions



confirmed on synthetic datasets

Conclusion

Contribution:

- ASKeD-BN: Automatic Synthesis of Boolean Networks constrained in their structure (PKN knowledge) and their dynamics (TS data)
- Approach free of strong / restraining assumptions
- Formulation as a logic program (Answer-Set Programming)
- ► ASKeD-BN gives good results

All data + code available at: https://gitlab.inria.fr/avaginay/OLA2021

Work in progress:

 Apply ASKeD-BN on PKN and TS directly extracted from existing biological models (ODE-like) The end. Any questions ?

Annexe

Datasets for Evaluation

2 real datasets:

Custom	PKN		TS	
System	# nodes	# edges	# time steps	# transitions
yeast (cell cycle)	4	28	14	6
A. thaliana (circadian clock)	5	8	50	11

6 synthetic datasets:

Various complexity: from 3 to 10 nodes.

Various conditions: synch. or async. update scheme, with or

without repetition, with or without noise

336 experiments at total including 42 with the ARN setting.

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336 experiments at total including 42 with the ARN setting.

Comparison time and RAM

yest					
method	running time (s)	cputime (s)	max_rss (MB)		
REVEAL	1.0095	0.55	72.97		
Best-Fit	1.4069	1.10	92.77		
caspo-TS	24.6545	12.91	183.08		
ASKeD-BN	5.4209	4.90	186.80		
A. thaliana					
method	running time (s)	cputime (s)	max_rss (MB)		
caspo-TS	7.0394	1.85	139.93		
ASKeD-BN	8.5820	8.19	163.38		

observed in general:

- caspo-TS faster and less RAM usage than ASKeD-BN
- but not on some datasets, where ASKeD-BN terminated in less than 100 hours while caspo-TS is still running after more than 300 hours. ← to investigate...

Synthetic Data – Comparison of the Number of BN Returned

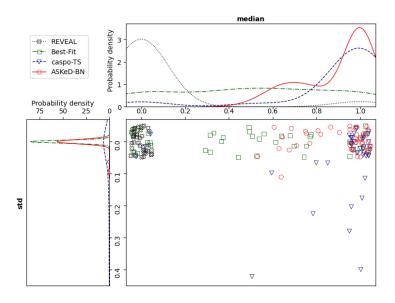
336 experiments at total including 42 with the ARN* setting.

		REVE	EAL	Best-	Fit	TC	ASKeD-BN	
	setting	before filter	after filter	before filter	after filter	caspo-TS		
# failing xp	all	230	240	0	64	20	0	
# total returned BNs	all	100 677 500	406	100 678 198	724	8481	1210	
# total returned BNs	ARN	3	3	51	35	720	85	

- ► REVEAL often fails
- REVEAL and Best-Fit return a lots of BNs which are not respecting the PKN
- caspo-TS returns in average between 5 and 7 times more BNs than ASKeD-BN (depending on the setting)

^{*:} ARN = Asyn. update scheme, with repetition and noise

Synthetic data, ARN setting – Quality of the BNs



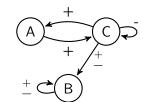
"A activates C"

"B interacts with itself"

"C activates A"

"C interacts with B"

 $\hbox{``C inhibits itself'}\\$



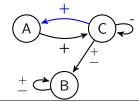
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For A:

3 choices:

A := C

A := 0

A := 1

but not:

A := B

 $\mathsf{A} := \neg \mathsf{C}$

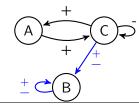
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For A: For B: $3 \text{ choices:} \\ A := C \\ A := 0 \\ A := 1 \\ \text{but not:} \\ A := B \\ A := \neg C$ For B: $16 \text{ choices:} \\ B := B \land \neg C \\ B := (B \land \neg C) \lor (\neg B \land C); \\ \Box B := 0 \\ B := 1 \\ \text{but not:}$

 $\mathsf{B} := \mathsf{A}$

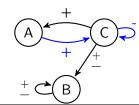
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For A:	For B:	For C:
3 choices: A := C A := 0	16 choices: $ B := B \land \neg C $ $ B := (B \land \neg C) \lor (\neg B \land C); $	6 choices: C := ¬C C := A
A := 1 but not: A := B	B := 0 B := 1	C := 0 C := 1
$A := \neg C$	but not: $B := A$	but not: $C := A \wedge B$

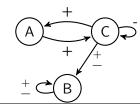
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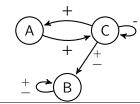
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ASKeD-BN: Soft constraints — Example 1

	\parallel 010 $ ightarrow$ 011 $ ightarrow$ 100 $ ightarrow$ 001																			
t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Α	0	3	7	13	20	30	49	61	100	63	36	25	2	3	1	1	3	0	0	0
В	100																	5	2	0
С	0	27	36	42	60	75	54	44	38	48	60	72	88	90	100	100	100	100	100	100

 \mathscr{U} set of unexplained timesteps Mean Absolute Error $\text{MAE}_{\mathsf{X}} = \frac{\sum_{t' \in \mathscr{U}} |\theta_{\mathsf{X}} - x_{t'}|}{T}$

$$\begin{array}{cccc} a_{t+1}=c_t & \checkmark & a_{t+1}=0 \\ \mathscr{U} & \emptyset & & \{8\} \\ \mathrm{MAE} & 0 & \checkmark & 0.55 \end{array}$$

ASKeD-BN: Soft constraints — Example 2

	\parallel 010 $ ightarrow$ 011 $ ightarrow$ 100 $ ightarrow$ 001																			
t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Α	0	3	7	13	20	30	49	61	100	63	36	25	2	3	1	1	3	0	0	0
	100																	5	2	0
C	0	27	36	42	60	75	54	44	38	48	60	72	88	90	100	100	100	100	100	100

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$$b_{t+1} = b_t \wedge \neg \mathsf{c}_t \quad \checkmark \quad b_{t+1} = (b_t \wedge \neg c_t) \vee (\neg b_t \wedge c_t)$$

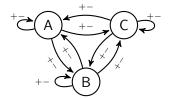
$$\emptyset$$

$$\mathsf{MAE} \qquad 0 \qquad \checkmark \qquad 0 \qquad \checkmark$$

$$\# \text{ influences} \qquad 2 \qquad \checkmark \qquad 4$$

How does the PKN help reducing the search space?

Without information



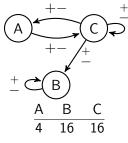
 $2^{2^3} = 256$ candidates for each components

 $\rightarrow 256 \times 256 \times 256 = 16777216$ candidate BNs

How does the PKN help reducing the search space?

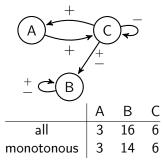
With PKN

directions only (REVEAL & Best-Fit)



ightarrow 4 imes 16 imes 16 = 1024 candidate BNs

direction + signs
(caspo-TS & ASKeD-BN)



 $\rightarrow 3 \times 16 \times 6 = 288$ candidate BNs including 252 locally partial-monotonous.